FTUV-09-1208

arXiv:0912.2172v2 [hep-ph] 20 Jan 2010

Gluonic components of the pion and the transition form factor $\gamma^*\gamma^* \to \pi^0$

N.I. Kochelev[a,b*] and V. Vento[b †]

(a) *Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, Moscow region, 141980 Russia*

(b)*Departament de Física Teòrica and Institut de Física Corpuscular, Universitat de València-CSIC, E-46100 Burjassot (Valencia), Spain*

Abstract

We propose an effective lagrangian for the coupling of the neutral pion with gluons whose strength is determined by a low energy theorem. We calculate the contribution of the gluonic components arising from this interaction to the pion transition form factor $\gamma^*\gamma^* \to \pi^0$ using the instanton liquid model to describe the QCD vacuum. We find that this contribution is large and might explain the anomalous behavior of the form factor at large virtuality of one of the photons, a feature which was recently discovered by the BaBar Collaboration.

Pacs: 12.38.Lg, 13.40.Gp
Keywords: pion, gluon component, instanton, form factor

*kochelev@theor.jinr.ru
†vicente.vento@uv.es

The implications, of recent data by the BaBar Collaboration [1] on the transition form factor $\gamma^*\gamma \to \pi^0$, in our understanding of the structure of the pion are being widely discussed [2, 3, 4, 5, 6]. A possible scenario to explain these data consists in assuming a flat shape for pion distribution amplitude [3, 4], supported by some low energy models like the Nambu-Jona-Lasinio model [7, 8] and instanton based non-local chiral quark model [9],and its detailed behavior under Quantum Chromodynamics (QCD) evolution, and a large mass cut-off added to the quark propagator [3], signalling a peculiar behavior of the $\overline{q}q$ wave function [10]. Never mind the various explanations, what appears evident is that the BaBar data[1], if confirmed, are in contradiction with most model predictions based on the factorization approach to exclusive reactions at large momentum transfer [11] and this apparent lack of perturbative factorization motivates the present investigation. In this Letter we suggest an alternative nonperturbative explanation to the BaBar results based on the existence of additional contributions to the pion form factor never previously considered. These contributions arise from the admixture of gluonic components, associated to nonperturbative properties of the QCD vacuum, which provide a strong interaction with two photons.

Let us propose a low-energy effective π^0 interaction with gluons of the following form

$$\mathcal{L}^{eff}_{\pi gg} = -\frac{1}{f_G^{\pi^0}}\pi^0\frac{\alpha_s}{8\pi}G^a_{\mu\nu}\widetilde{G}^a_{\mu\nu}. \tag{1}$$

Such type of Lagrangian density, describing the interaction of a pseudoscalar meson with gluons, was introduced many years ago by Cornwall and Soni [12] to derive Witten's relation between the η' mass and the topological susceptibility, in a world without light quarks [13]

$$\chi_t^{N_f=0} = -\frac{f_\pi^2}{6}(M^2_{\eta'} + M^2_\eta - 2M^2_K), \tag{2}$$

where $f_\pi = 92.3$ MeV is the pion decay constant, M_X the mass of the indicated particles, the topological susceptibility is given by

$$\chi_t^{N_f=0} = i\int d^4x < 0|T\{Q_5(x)Q_5(0)\}|0 >_G, \tag{3}$$

and

$$Q_5(x) = \frac{\alpha_s}{8\pi}\, G^a_{\mu\nu}(x)\widetilde{G}^a_{\mu\nu}(x) \tag{4}$$

is the topological charge density.

The effective pion-gluon interaction, Eq.1, is the analogue of the pion-quark effective interaction

$$\mathcal{L}^{eff}_{\pi qq} = -\frac{1}{f_\pi}M_q\bar{q}i\gamma_5\vec{\tau}q\cdot\vec{\pi}, \tag{5}$$

giving the pion coupling to the quarks.

To derive the decay constant $f_G^{\pi^0}$, which sets the scale of the gluon nonperturbative interaction with the neutral pion, we will use a low energy theorem (LET) [14]

$$< 0|\frac{\alpha_s}{8\pi}G^a_{\mu\nu}\widetilde{G}^a_{\mu\nu}|\pi^0 >= \frac{1}{2}\frac{m_d - m_u}{m_d + m_u}f_\pi M^2_\pi. \tag{6}$$

We stress that this matrix element is rather big due to the large light quark mass ratio [15]

$$z = \frac{m_u}{m_d} = 0.35 - 0.6. \tag{7}$$

By using the effective interaction Eq.1 and the LET Eq.6 we get

$$f_G^{\pi^0} = -\frac{2(1+z)}{(1-z)} \frac{\chi_t^{N_f=0}}{f_\pi^2 M_\pi^2} f_\pi. \tag{8}$$

As it was to be expected, the strength of the coupling of the neutral pion to gluons is related to the violation of isospin symmetry and proportional to the difference of the d- and u-quark masses

$$\frac{1}{f_G^{\pi^0}} \propto m_d - m_u. \tag{9}$$

For the value of the mass ratio m_u/m_d shown in Eq.7 which is the one allowed by the Particle Data Group, one obtains

$$R = \frac{f_G^{\pi^0}}{f_\pi} \simeq 28.1 - 51.1. \tag{10}$$

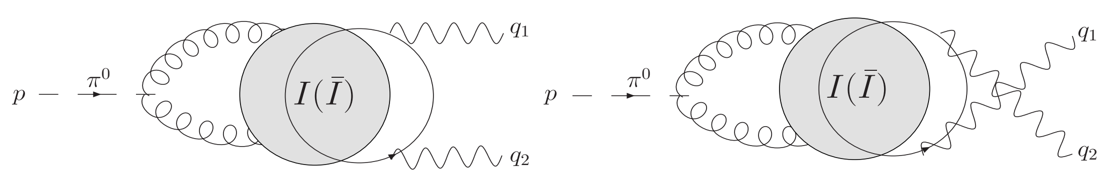

Figure 1: Gluonic contribution to the pion transition form factor. Symbol I ($\bar{I}$) denotes instanton(antiinstanton).

Let us next calculate the contribution arising from the interaction given by Eq.1 to the transition form factor of two photons to the pion $\gamma^*(q_1)\gamma^*(q_2) \to \pi^0(p)$, where q_1 and q_2 are the photon momenta and $q_1 + q_2 = p$. We consider the case where all virtualities of the incoming and the outcoming particles are in the Euclidean domain $Q_1^2 = -q_1^2 \geq 0$, $Q_2^2 = -q_2^2 \geq 0$, $P^2 = -p^2 \geq 0$. The Lagrangian density Eq.1 describes the pion interaction with soft gluons. Such gluons should interact with photons through nonperturbative QCD interactions. We use the instanton liquid model (ILM) for the QCD vacuum to calculate the interaction of two photons with the gluonic component of the pion. The ILM is one of the most successful models for the description of nonperturbative QCD effects (see reviews [16, 17]). Within the ILM the single instanton contribution to the pion form factor coming from the interaction Eq.1 is associated to the diagrams in Fig.1.

The amplitude for the $\pi^0-\gamma^*\gamma^*$ interaction via an instanton with center z_0 has the following form

$$\begin{aligned}
T_{\mu\nu}(p,q_1,q_2) &= \epsilon_{\mu\nu\alpha\beta}\sum_i e_i^2\frac{4}{f_G^{\pi^0}\pi^4}\int n_I(\rho)\rho^2 d\rho \times \\
&\int d^4z_0\int d^4z_1\int d^4x\int d^4y e^{-ipz_1}e^{iq_1x}e^{iq_2y}Q_5^I(\bar z_1))\times \\
&\frac{h_{\bar x}h_{\bar y}}{\Delta^2}\left\{\frac{1}{\Delta^2}(h_{\bar y}\Delta^\alpha\bar y^\beta-h_{\bar x}\Delta^\beta\bar x^\alpha)+h_{\bar x}h_{\bar y}\bar x^\alpha\bar y^\beta\right\}, \qquad (11)
\end{aligned}$$

where

$$Q_5^I(\bar z_1))=\frac{6\rho^4}{\pi^2(\bar z_1^2+\rho^2)^4} \qquad (12)$$

is the topological charge density of the instanton in the space-time point z_1, $n_I(\rho)$ is the instanton density, ρ is the instanton size, $\Delta=\bar x-\bar y$, $h_{\bar x}=1/(\bar x^2+\rho^2)$, $h_{\bar y}=1/(\bar y^2+\rho^2)$ and the notation $\bar w\equiv w-z_0$ for any variable w has been introduced. The sum runs over the light quark flavors, i.e. $i=u,d,s$. To get Eq.11 we have used the correlator of two electromagnetic currents in the instanton field obtained by Andrei and Gross [18]. Their result was corrected by a color factor (see [19]). The first term in the last line of the equation is coming from the quark nonzero modes in the instanton field and the last term arises from the interference between nonzero and zero modes [18].

The final result for the gluon contribution to the pion transition form factor induced by instantons is

$$F(P^2,Q_1^2,Q_2^2)_g^I=\frac{4<e^2>}{f_\pi R}\int d\rho n_I(\rho)\rho^4 S(\rho,P^2,Q_1^2,Q_2^2) \qquad (13)$$

where

$$S(\rho,P^2,Q_1^2,Q_2^2)=\Phi_1(\sqrt{z_3})\int_0^1 dt\left\{I(t,z_1,z_2,z_3)+(1-t)I(t,z_2,z_1,z_3)\right\}, \qquad (14)$$

$$I(t,z_1,z_2,z_3) = \int_0^\infty d\alpha\frac{\alpha(\alpha+1)\Phi_2(Z(\alpha,t,z_1,z_2,z_3))}{(\alpha+1-t)^3Z^2(\alpha,t,z_1,z_2,z_3)}, \qquad (15)$$

and

$$Z(\alpha,t,z_1,z_2,z_3)=\sqrt{(\alpha+1)(t\alpha z_1+tz_2+(1-t)z_3)/(\alpha+1-t)}. \qquad (16)$$

The functions

$$\Phi_1(z)=\frac{z^2K_2(z)}{2},\ \Phi_2(z)=zK_1(z) \qquad (17)$$

behave as $\Phi_{1,2}(z)\to 1$ in the limit $z\to 0$. In Eqs.13-16 the notations are $z_1=Q_1^2\rho^2$, $z_2=Q_2^2\rho^2$, $z_3=P^2\rho^2$ and $<e^2>=\sum_i e_i^2$.

For an estimate we use Shuryak‘s version of the ILM [20], where the density is given by

$$n_I(\rho)=n_0\delta(\rho-\rho_c) \qquad (18)$$

and

$$n_0\approx 1/2fm^{-4},\quad \rho_c\approx 1/3fm. \qquad (19)$$

Within this simple model for the instanton distribution the result for the form factor is

$$F(P^2, Q_1^2, Q_2^2)_g^I = \frac{4 < e^2 > f_I}{\pi^2 f_\pi R} S(\rho_c, P^2, Q_1^2, Q_2^2), \tag{20}$$

where $f_I = \pi^2 n_0 \rho_c^4$ is so-called instanton packing fraction in the QCD vacuum.

It should be pointed out that in spite of the smallness of instanton packing fraction $f_I \approx 0.06$, using the single instanton approximation as above is only valid for values of the momentum transfers $Q_1, Q_2 \gg 1/R_I$, where $R_I \approx 3\rho_c$ is the distance between the instantons in the ILM. For smaller photon virtualities it is necessary to include the contributions arising from multiinstanton configurations. With an average size of the instanton in the QCD vacuum as in Eq.19 for the region $Q_1^2, Q_2^2 \geq 1/\rho_c^2 \geq \mu^2 = 0.35$ GeV^2, i.e. $z_{1,2} \geq 1$, the validity of a single instanton approximation is assured.

The calculation above was done for the case when all external momenta are Euclidean. In order to compare with BaBar data we have to perform an analytic continuation of the pion virtuality to the physical point of the pion on-shell $P^2 \to -m_\pi^2 - i\epsilon$. An inspection of the integrals in Eqs.14,15 shows that the dominant contribution to the form factor at $m_\pi^2/Q_{1,2}^2 \ll 1$ is coming from the region of integration $t \approx 0$, due to the pole at $Z^2 = 0$. Assuming the following behavior of the function $\Phi_2(Z) \equiv Z K_1(Z) \sim 1$ near the pole and keeping only leading terms in m_π^2 we obtain following closed form formulas for the real and imaginary parts of the flavor singlet part of form factor

$$\begin{aligned}
Re(F(m_\pi^2, Q_1^2, Q_2^2)_g^I) \simeq\ & \frac{4 < e^2 > f_I}{\pi^2 f_\pi R} \times \\
& \{ \frac{z_1[z_2 log(z_2)/z_1 + log(z_1)(log(z_1/z_2) - 1) + Li_2((z_1 - z_2)/z_1)]}{(z_1 - z_2)^2} + \\
& \frac{z_2[z_1 log(z_1) - \pi^2/6 - log^2(z_1 - z_2)/2 - log(z_2)(1 - log(z_1)/2)]}{(z_1 - z_2)^2} + \\
& \frac{z_2[Log(z_2) log((z_1/(z_1 - z_2)) - Li_2(z_2/(z_2 - z_1))]}{(z_1 - z_2)^2} - \\
& \frac{log(z_1/z_2) log(m_\pi^2 \rho_c^2)}{z_1 - z_2} \},
\end{aligned} \tag{21}$$

$$Im(F(m_\pi^2, Q_1^2, Q_2^2)_g^I) \simeq \frac{4 < e^2 > f_I}{\pi f_\pi R} \frac{log(z_1/z_2)}{z_1 - z_2}. \tag{22}$$

The imaginary part of form factor arises because the pion may decay in this calculation into a quark-antiquark pair since confinement, which forbids this decay, is not explicitly implemented. However, the net contribution of the imaginary part to total transition form factor in the BaBar kinematics is very small. These formulas are useful to extract the behavior of the transition form factor with Q^2. The exact numerical analysis will be described below. For definiteness we consider the case $z_1 > z_2$

In the limit $Q_1^2 \gg Q_2^2$ which is valid for BaBar kinematics, the formulas for the real and the imaginary parts simplify,

$$Re(F(m_\pi^2, Q_1^2, Q_2^2))_g^I \approx \frac{4 < e^2 > f_I}{\pi^2 f_\pi R} \frac{[log(Q_1^2/m_\pi^2) log(Q_1^2/Q_2^2) + \pi^2/6]}{\rho_c^2 Q_1^2}, \tag{23}$$

$$Im(F(m_\pi^2, Q_1^2, Q_2^2))_g^I \approx \frac{4 < e^2 > f_I}{\pi f_\pi R} \frac{log(Q_1^2/Q_2^2)}{\rho_c^2 Q_1^2}. \tag{24}$$

It follows from Eqs.23,24 that the flavor singlet gluon induced part of the form factor has a dependence on the large photon virtuality Q_1^2 proportional to $log^2(Q_1^2)/Q_1^2$, which is much stronger than that of the flavor nonsinglet part, which in most of the models is of the form $1/Q_1^2$. The additional feature of this new contribution is its strong chiral enhancement since the massless logs appear governed by the pion mass as $log(Q_1^2/m_\pi^2)$. For symmetric kinematics $Q^2 = Q_1^2 = Q_2^2$ the result is

$$Re(F^S(Q^2))_g^I \approx \frac{< e^2 > f_I}{\pi^2 f_\pi R}\frac{(3 + 2log(Q^2/m_\pi^2))}{\rho_c^2 Q^2}, \tag{25}$$

$$Im(F^S(Q^2))_g^I \approx \frac{2 < e^2 > f_I}{\pi f_\pi R \rho_c^2 Q^2}. \tag{26}$$

Having determined the dependence of the virtuality in the approximation Eq. 21, we proceed to study the exact numerical calculation which includes the effect of the functional form of $ZK_1(Z)$. Before we do so, we would like to point out that the exact calculation leads to a smaller (by about 50%) result, compared to the approximate calculation. This factor can be absorbed in the uncertainties of the vacuum model associated with the poor knowledge of the instanton distribution (about 30%), and the additional uncertainty coming from the value of the pion coupling to gluons Eqs.9 and 10 (about a factor 2) due indeterminacy in the ratio of u- and d- quark masses, Eq.7.

We compare our result with the BaBar data. Before we do so some caveats have to be expressed since the comparison is not direct. The BaBar experiment, only measures the virtuality of one of the photons in the interval $Q_1^2 = 4 - 40$ GeV^2. They only put an upper limit on the virtuality for the second photon, $Q_2^2 < 0.18$ GeV^2. Finally, they use a model for the form factor to extract the value at the real photon point $Q_2^2 = 0$. Thus, a direct comparison of our results with the BaBar data is not possible. Moreover, our calculation only represents the flavor singlet contribution to the form factor, therefore we have to add a flavor nonsinglet part. We take in the estimate shown in Fig.2 for the flavor nonsinglet part the corresponding to a vector meson dominance (VMD) model, i.e.

$$F(Q_1^2, Q_2^2)_q^{VMD} = \frac{1}{4\pi^2 f_\pi}\frac{1}{(1 + Q_1^2/M_\rho^2)(1 + Q_2^2/M_\rho^2)}. \tag{27}$$

In order to compare our results with the BaBar data we perform an extrapolation of their results from $Q_2^2 = 0$ to $Q_2^2 = 0.35$ GeV^2. In Fig.2 we compare our calculation with the extrapolation of the BaBar data described by [1]

$$Q^2 \mid F_{exp}^{BaBar}(Q^2) \mid = A\left(\frac{Q^2}{10GeV^2}\right)^\beta, \tag{28}$$

where $A \simeq 0.182$ and $\beta \simeq 0.25$. This function has been continued to the point μ^2(0.35 GeV^2 in our case) following the VMD model,

$$Q^2 \mid F^{BaBar}(Q^2, \mu^2) \mid = \frac{Q^2 \mid F_{exp}^{BaBar}(Q^2) \mid}{1 + \mu^2/M_\rho^2}. \tag{29}$$

The bands in the figure represent our uncertainties, both in the vacuum model and in the coupling constant, as mentioned before.

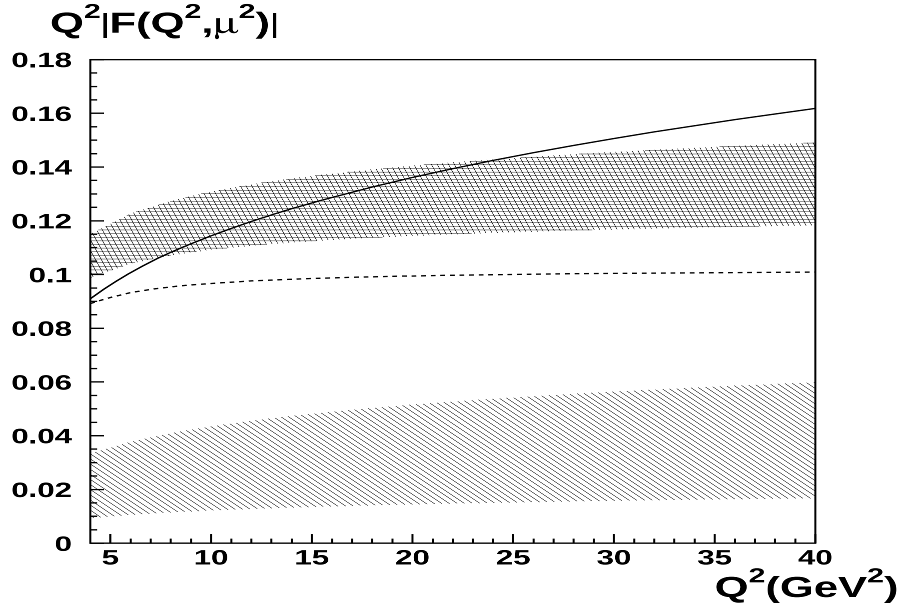

Figure 2: Contributions to the pion transition form factor compared with the extrapolation of the BaBar data to $Q_2^2 = \mu^2 = 0.35$ GeV^2 (solid line): gluonic contribution in our model including uncertainties (lower band), conventional VMD contribution (dashed line) line, and their sum (upper band).

It should be mentioned that the behavior of the gluonic part of the form factor as function of Q^2 is determined by shape of the decay of the non-zero modes in the instanton field (see Eq.11). At the same time it is well known that the VMD-like behavior of the flavor nonsinglet part of pion form factor can be easily reproduced within a non local chiral quark model based on the quark zero-modes dominance in the instanton vacuum[21]. Due to the weaker decay of the quark nonzero modes with respect to zero modes one can expect a harder Q^2 dependence of the flavor singlet part of the form factor in comparison with the flavor nonsinglet part. Such tendency is seen in Fig.2. Indeed, the gluonic part is described well by a fast increasing function of Q^2, Eq.23. That function shows a similar behavior, as function of Q^2, as the BaBar data, Eq.28. Contrary to the gluonic part of the form factor, its flavor nonsinglet part has a conventional $1/Q^2$ behavior at large Q^2, Fig.2. Taking into account some uncertainties in our estimates related to the poorly known ratio of the u- and d- quark masses, Eq.7, as well as uncertainties in the parameters of the instanton model, we conclude, that the new contribution related to the gluonic component of pion might explain the anomalous behavior of the pion transition form factor found by the BaBar Collaboration. One should be aware that these contributions are beyond the Operator Product Expansion and their QCD evolution is non trivial [22].

It is evident that such type of contribution should be present also for the η and η' mesons. In this case one should carefully take into account the effects of their larger masses and their strong mixing (see recent discussion in the papers [23, 24]). We also would like to point out that the strong Q^2 dependence of the gluonic part of the form factor opens a new possibility to disentangle particles with dominant gluonic content, i.e. glueballs (see review [25]), in $\gamma^*\gamma^*$ collision. These tasks are the subject of future investigations.

In summary, the BaBar data [1] point towards a breaking of perturbative factorization.

This has led us to investigate possible non perturbative mechanism that contribute to the pion transition form factor. We have shown that a nonzero interaction of neutral pion with gluons arising from isospin violation, i.e. $m_u \neq m_d$, induces a large contribution to this form factor at large virtuality of one of the photons. More sophisticated models for the instanton density and probably multiinstanton contribution might bring the value closer to the observed one.

Acknowledgments

We are grateful to A.E. Dorokhov and S. Noguera, for very useful discussions. NIK thanks the Departament de Física Teòrica Universitat de València for the hospitality and the University of Valencia for a Visiting Professor appointment. This work was supported in part by HadronPhysics2, a FP7-Integrating Activities and Infrastructure Program of the European Commission under Grant 227431, by the MICINN (Spain) grant FPA2007-65748-C02-01, by GVPrometeo2009/129, by the RFBR grant 10-02-00368-a and by Belarus-JINR grant.

References

[1] B. Aubert *et al.* [The BABAR Collaboration], Phys. Rev. D **80** (2009) 052002.

[2] A. E. Dorokhov, arXiv:0905.4577 [hep-ph]; arXiv:0909.5111 [hep-ph].

[3] A. V. Radyushkin, arXiv:0906.0323 [hep-ph].

[4] M. V. Polyakov, JETP Lett. **90** (2009) 228 [arXiv:0906.0538 [hep-ph]].

[5] S. V. Mikhailov and N. G. Stefanis, Nucl. Phys. B **821** (2009) 291 [arXiv:0905.4004 [hep-ph]]; arXiv:0910.3498 [hep-ph].

[6] V. L. Chernyak, arXiv:0912.0623 [hep-ph].

[7] E. Ruiz Arriola and W. Broniowski, Phys. Rev. D **66** (2002) 094016 [arXiv:hep-ph/0207266].

[8] A. Courtoy and S. Noguera, Phys. Rev. D **76** (2007) 094026 [arXiv:0707.3366 [hep-ph]].

[9] I. V. Anikin, A. E. Dorokhov and L. Tomio,
Phys. Lett. B **475** (2000) 361 [arXiv:hep-ph/9909368].

[10] S. Noguera and V. Vento, work in preparation.

[11] G. P. Lepage and S. J. Brodsky, Phys. Lett. B **87** (1979) 359; Phys. Rev. D **22** (1980) 2157.

[12] J. M. Cornwall and A. Soni, Phys. Rev. D **32** (1985) 764.

[13] E. Witten, Nucl. Phys. B **156** (1979) 269.

[14] D. J. Gross, S. B. Treiman and F. Wilczek, Phys. Rev. D **19** (1979) 2188.

[15] C. Amsler et al. (Particle Data Group), Phys. Lett. B **667** (2008) 1, and 2009 partial update for the 2010 edition.

[16] T. Schäfer and E.V. Shuryak, Rev. Mod. Phys. **70** (1998) 1323.

[17] D. Diakonov, Prog. Par. Nucl. Phys. **51** (2003) 173.

[18] N. Andrei and D. J. Gross, Phys. Rev. D **18** (1978) 468.

[19] E. V. Shuryak, Phys. Rept. **115** (1984) 151.

[20] E. V. Shuryak, Nucl. Phys. B **203** (1982) 93;116;140.

[21] A. E. Dorokhov, JETP Lett. **77** (2003) 63 [Pisma Zh. Eksp. Teor. Fiz. **77** (2003) 68]

[22] A. V. Manohar, Phys. Lett. B **244** (1990) 101.

[23] V. Mathieu and V. Vento, arXiv:0910.0212 [hep-ph].

[24] Y. N. Klopot, A. G. Oganesian and O. V. Teryaev, arXiv:0911.0180 [hep-ph].

[25] V. Mathieu, N. Kochelev and V. Vento, Int. J. Mod. Phys. E **18** (2009) 1 [arXiv:0810.4453 [hep-ph]].